Due: 16h00, 16 Feb 2018 Homework 2

PH4204: High Energy Physics

1. **Kinematics of emission and absorption:** Consider a model (QED) of an electron with mass m_e interacting with a massless photon. In this model consider the processes of emission and absorption of photons:

$$e^-(p_i) \to e^-(p_f) + \gamma(k) \ \ (emission) \qquad\qquad e^-(p_i) + \gamma(k) \to e^-(p_f) \ \ (absorption)$$

Show that when initial (p_i) and final (p_f) electrons are on-mass-shell ($p_i^2 = m_e^2 = p_f^2$) then the emitted and absorbed photon cannot be on-mass-shell. Also show that only two (or less) of the three particles can be on-mass-shell at a given time.

2. **Kinematics of reaction and decay:** Consider a scattering process with two colliding beams $B_{1,2}$ going into two final stat particles $A_{1,2}$ as:

$$B_1(m, k_1) + B_2(m, k_2) \to A_1(m_1, p_1) + A_2(m_2, p_2)$$

Here k_i and p_i are four momenta of the particles and m, m_i are the masses of the particles. Assume center of momentum frame and the total energy in this frame to be $E_{cm}(> m_1 + m_2)$.

   (a) Find the energies $E_{1,2}$ for the final state particles $A_{1,2}$ in terms of masses and E_{cm}.

   (b) Find the momentum $|\vec{p_1}| = p = |\vec{p_2}|$ in terms of the masses and E_{cm}.

   (c) Consider a decay $B(M, k) \to A_1(m_1, p_1) + A_2(m_2, p_2)$ with similar final state as above reaction in the rest frame of B. Assuming $M > m_1 + m_2$ find the similarity and difference in the final state kinematical variables between this decay and above reaction.

3. **QED with two fermions:** Consider the QED Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}_1 (i \not{D} - m_1) \psi_1 + \bar{\psi}_2 (i \not{D} - m_2) \psi_2$$

where $\psi_{1,2}$ is a fermion having $Q_{1,2}$ charge in unit of e, the coupling constant.

   (a) Write down the Feynman rules for the above Lagrangian.

   (b) Show that the *fermion type number* (similar to Lepton number of electron type) is separately conserved for the two fermions ψ_1 and ψ_2.

   (c) Consider the *fermion type number* violating decay $\psi_2 \to \psi_1 \gamma$ ($m_2 > m_1$). Using graph theory (and Wick's theorem), show that the amplitude for this process remains zero at all order in the perturbation theory.

4. **QED and Bhabha scattering:** Consider the QED Lagrangian given in previous problem. Assuming the label 1 for the electrons and 2 for muons, consider the Bhabha's scattering

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \to e^-(p_3, \lambda_3) + e^+(p_4, \lambda_4)$$

where p_i's are the four momenta and λ_i's are the helicities.

   (a) Using the Feynman rules in the previous problem, draw all the diagrams and write the expreson for each of them.

   (b) Calculate the matrix element square after summing over initial and final state helicities. (*Hint: You may use the FORM program to calculate the trace.*).

5. **QED and Compton scattering:** Consider the QED Lagrangian given in problem 3. Assuming the label 1 for the electrons and 2 for muons, consider the Compton scattering

$$e^-(p_i, \sigma_i) + \gamma(k_i, \lambda_i) \to e^-(p_f, \sigma_f) + \gamma(k_f, \lambda_f)$$

where p_i and k_i are the four momenta and σ_i and λ_i are the helicities of electron and photons, respectively.

(a) Draw all the Feynman diagrams for the above process and write down the matrix elements.

(b) Using the identity $\sum_\lambda \epsilon^*_\mu(k,\lambda)\epsilon_\nu(k,\lambda) = -g_{\mu\nu}$ for the photon polarization sum, calculate the matrix element square for the process.
(*Hint: You should use the FORM program to calculate the trace of a string of 8* γ^μ *matrices.*).